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# Xplora Thematic Dossier: Fractals

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**Every month, Xplora will release a new thematic dossier on an important topic in science, complete with background information, recommended resources, experiments, training opportunities and more.**

**This month the focus are Fractals – it includes top tips for using software to create fractals and more.**

Many of us have admired natural phenomena like the curve of a sea shell or the shape of a fern. Most natural forms can be described mathematically in terms of fractals (Link to http://en.wikipedia.org/wiki/Fractal (English), http://de.wikipedia.org/wiki/Fraktal (Deutsch)

http://fr.wikipedia.org/wiki/Fractal (Francais)). The beauty of the calculated objects is so overwhelming, that it is easy to motivate students to learn about complex numbers, formal languages or vector functions, depending on the type of fractals you are going to use. You will find information about the following three fractal types later on:

- Escape time fractals
- Lindenmayer systems (L-systems)
- Iterated Function Systems (IFS)

Every type of fractal offers a pathway to a different topic of mathematics, with one suitable for even lower secondary computer science education.

A fractal is a mathematical object, which contains in a visual representation the property of self similarity.  The term fractal was coined in 1975 by Benoît Mandelbrot, from the Latin *fractus* or "broken". Aside the fact that the object looks fractioned, the term fractal was given due to the strange fact that these objects have a dimension which is not an whole number but a fraction. Before Mandelbrot coined this term, the common name for such structures (e.g. the Koch snowflake) was monster curve.

The euclidean dimension of an object is (simplified) the number of coordinate axes, needed to display all points on it. All points on a line will need only one coordinate to be shown, hence the dimension of a line is dim(line)=1. A planar triangle needs two coordinates to show all points in it. Hence the dimension of a planar triangle is dim(triangle)=2.  For a cube the dimension is accordingly dim(cube)=3.

This simple definition no longer holds with fractals. The full theory of the Hausdorff dimension is probably too complicated for schools, but there is an easy way of explaining the results of the Hausdorff dimension of fractals like the Koch curve, which returns the Euclidian dimensions for Euclidian objects.

According to this theory, the Hausdorff dimension can be defined as the quotient of the logarithms of any base of the number of elements needed to build the new self similar object and the resulting length of the object.

$$dim(o)=\frac{\log(n)}{\log(l)}$$

*Equation 1: The simple version of the Hausdorff dimension. O is the object, n is the number of objects needed to build the next selfsimilar object, l is the length of the new object.*

The next image shows the application of the above definition to simple Euclidean objects and the Koch fractal. For Euclidean objects the dimensions are the well known numbers, while for the Koch fractal the dimension is a fraction of real numbers, a fact which gave these objects the name "fractals".

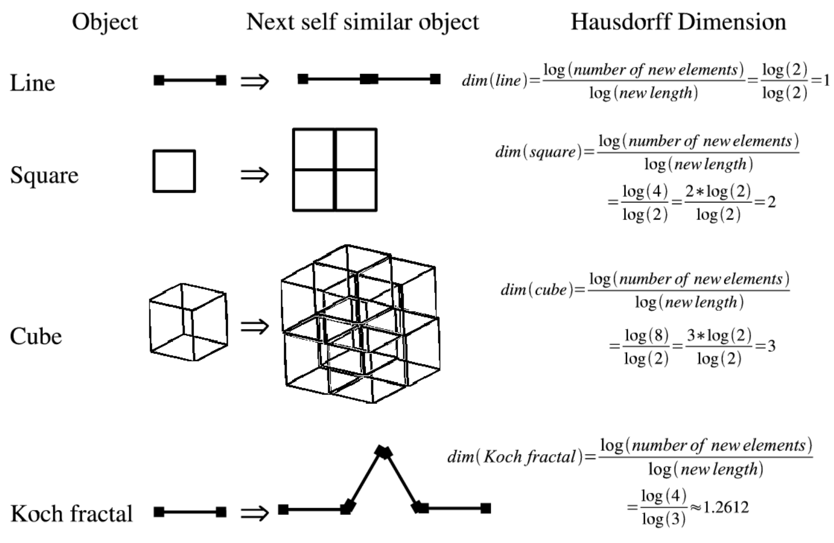


*Figure 1: The Hausdorff dimension of simple euclidean objects gives the well known values, while the dimension of fractal objects is a fraction of real numbers.*

The name *Escape time fractals* results from the algorithm used for calculation. The Mandelbrot set is the most well known example of this kind of fractal.

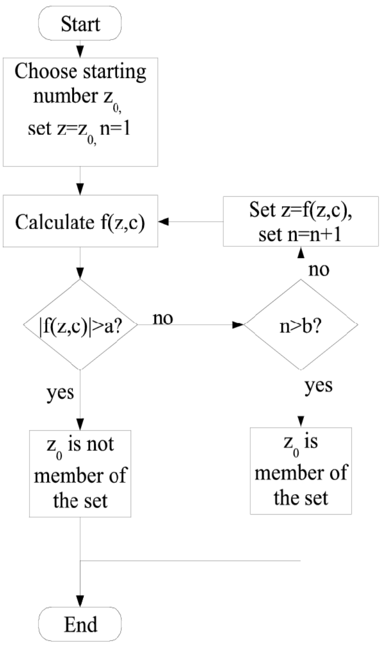


*Figure 2: The data flow for the calculation of escape time fractals.*

A given complex number $z_0$ is tested to see if it belongs to a certain set by calculating the function value $z=f(z_0,c)$, where c is a second complex number, which depends on the kind of set you want to investigate. If the absolute value of z (distance from the origin) exceeds the value a (the bail out value), the number $z_0$ is assumed not to be part of the set under investigation.  Otherwise the number of iterations n is checked. If the number of iterations is less than a given threshold b, the number z is replaced by the function value f(z,c) and the iteration procedure continues. If the number of iterations exceeds the iteration threshold b, the number $z_0$ is assumed to be part of the investigated set.

For the famous Mandelbrot set, we have the function  $f(z,c)=z^2+c$ and $c=z_0$.

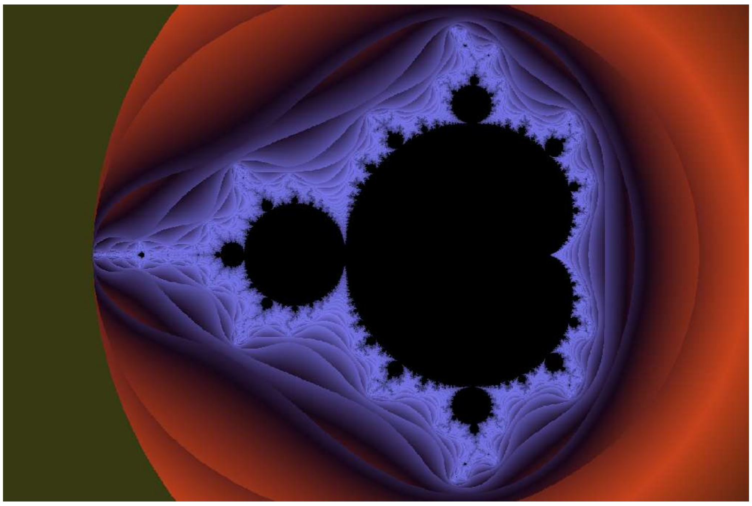

*Figure 3: The famous Mandelbrot set is drawn in black in the center of the image. From an aesthetic point of view the regions at the border outside the set are more fascinating. Xaos allows easily to zoom in.*

The name L-system was given to the fractals to the honours of Aristid Lindenmayer, Hungarian biologist who created a formal language grammar which enables the calculation of fractal plant-like objects. The definition of an L-System is normally given as a set:

G = {V, S, ω, P},

where

- V (the alphabet) is a set of symbols containing elements that can be replaced (variables)
- S is a set of symbols containing elements that remain fixed (constants)
- ω (start, axiom or initiator) is a string of symbols from V defining the initial state of the system
- P is a set of rules or productions defining the way variables can be replaced with combinations of constants and other variables. A production consists of two strings - the predecessor and the successor.

According to this definition, the famous Koch curve is described by the following set G:

G={V={F}, S={+,-}, ω=F, P={F -> F+F–F–F+F}}

Using turtle graphics (like in LOGO), F is interpreted as a step Forward (e.g. Fd 100) in the actual turtle direction. The + sign is interpreted as a “turn left 60°” (lt 60) and the – sign is interpreted as “turn right 60°” (rt 60).

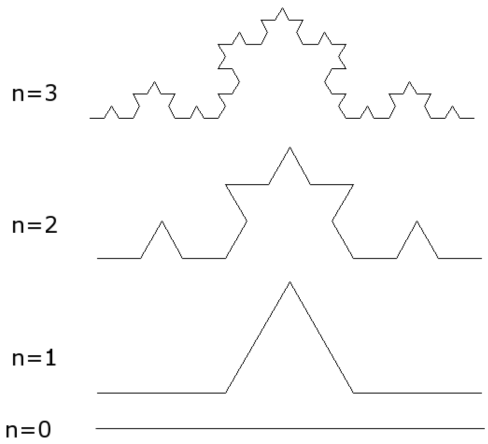


*Figure 4: Repeating the transformations in the grammar of the Koch curve gives the fractal coastline.*

If you are interested in teaching LOGO, translating L-systems into LOGO programs might be a wonderful experience for pupils of lower secondary classes. You might think of arranging a local exhibition of your pupils printed works after finishing the programming lessons, best at the end of a school term.

In IFS fractals, a set of functions is applied iterating on a set of points in the plane or space. The fractal is formed by the fixed points of the set of iterating functions. The individual functions are applied randomly by a given probability.

$$\vec{f}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} * \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix}$$

*Equation 2: Functions of the IFS in Fractint are given by this formula. The functions are applied at each iteration step with a predefined probability p*

The famous Barnsley fern uses the following functions in the program Fractint:

| a | b | c | d | e | f | p |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | .16 | 0 | 0 | .01 |
| .85 | .04 | -.04 | .85 | 0 | 1.6 | .85 |
| .2 | -.26 | .23 | .22 | 0 | 1.6 | .07 |
| -.15 | .28 | .26 | .24 | 0 | .44 | .07 |

This definitions implies the use of 4 different functions with probabilities ranging from p=0.01 to p=0.85.

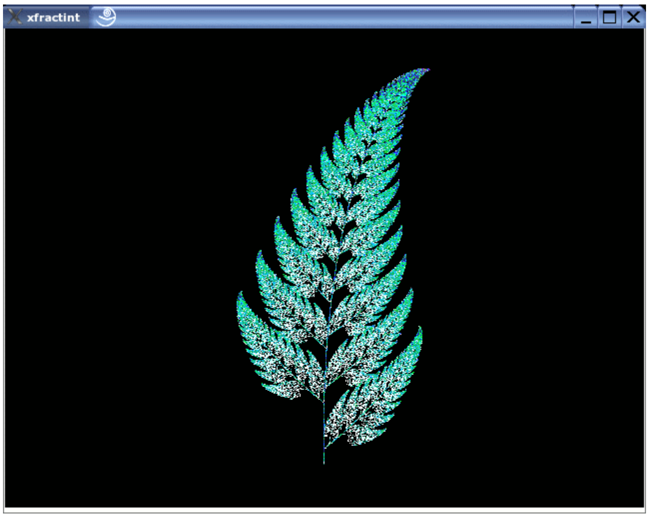


*Figure 5: The Barnsley fern as computed in xfractint.*

# 1. Resources

*Fractals and games about chaos – German only*
A complete course on IFS-type fractals in German language. This course requires some mathematical skills, so it is for pupils in upper secondary classes.
http://www.matheprisma.uni-wuppertal.de/Module/Fraktal/index.htm

*Fractals: An Introductory Lesson – English language*

An introduction to L-system fractals in English language. Only low level of mathematics is required to follow this introduction.

http://ejad.best.vwh.net/java/fractals/

*An approach to the world of fractals in secondary level entry classes – French language*

The PDF document in French language contains an article about using CABRI, a dynamic geometry software package, and FRACTINT (see below) in the classroom. It was introduced to pupils of secondary entry classes in the years of 1994-95.

http://icosaweb.ac-reunion.fr/GeomJava/abraCAda/Docs/lycee/abra11_2.pdf

*From chaos to fractals – in French*

This website covers topics from chaos to fractals. The pages contains tasks for students. The solutions of the tasks are given.

http://www.crdp.ac-grenoble.fr/imel/delahaye/chaos/infogene.htm

*Gallery of fractal images – French language*

The College G. Pompidou presents a rather short introduction to the mathematics of fractals. There is an amazing gallery of fractal images.

http://www.ac-versailles.fr/etabliss/clg-pompidou-orgerus/Galerie_0304/Fractales/Explications.htm

*Recursion and Sierpinski triangle – in German*

This is a students report from a lesson about recursion and the Sierpinski triangle. The developed JAVA computer program is described and visitors can test it directly on the page.

http://www.fh-niederrhein.de/~gkorsch/if1100/mvkm/stundenprotokoll.html

*Fractals challenge – in English*

Students from "Duiliu Zamfirescu" School in Focsani, Romania (age 14-15 years) report about their work. The main topics are the Peano curve and the Koch snowflake.

http://www.orillas.org/math/fractals.html

*Witnesses in the Middle of Nature – in English*

A report about fractals in LOGO from students of School No.10 Focsani, Romania (age 13-14 years). Besides some basic information about fractals, a gallery of artwork from pupils is available. The artwork was created using LOGO.

http://www.orillas.org/math/19971998/fract.html

*Commercial gallery of fractals – English language*

A selection of fractals is shown, ready to order. Ordering prints from this source supports the development of XaoS, an open source program for the calculation and visualisation of fractals. The prices are low and the author offers to print a text on the high quality image, which the students might send in. This can be a nice decoration for the classroom after a fractal project.

http://wmi.math.u-szeged.hu/~kovzol/particio/Fractals.php

*XaoS software - Multilingual*

The XaoS fractal creation program. It is available in versions for Windows, Mac OS X and Linux. Tips of how to compile a Mac OS X version are offered. Excellent tutorial material comes with the program. It is already contained in many Linux distributions. The tutorials are available in English, French, Czech, German, Spanish and Hungarian language.

http://xaos.sourceforge.net/english.php

*Fractal geometry – Chaos and order – in German language*

A web site with a lot of material about chaos and fractals. It includes a very interesting publication of an email discussion with experts about a definition of the term "fractal". The topics include the whole spectrum of theory, available software and how to use chaos and fractals in the classroom. It is worth noting that they use a Computer Algebra System (CAS) for the calculation of fractals.

http://www.uni-lueneburg.de/mathe-lehramt/fraktale/fraktale.htm

*Fractint software – English language*

This web page offers the download of Fractint, a free fractal generating program available for Windows, Mac and Linux. The web page contains links to a fractal contest, where users of Fractint can submit their creations.

http://spanky.triumf.ca/www/fractint/fractint.html

*Fractalwelt – German and most parts in English*

This site is a very rich resource about fractals. Many pages are available in English and German language, but most pages are in German language. There is software running under Windows only, but also a lot of JAVA programs with source code, which can run on any computer system with JAVA installed. An excellent gallery gives inspiration for work on beautiful fractals. There is a link to a web ring of websites dealing with fractals too.

http://www.fraktalwelt.de/

*JavaView – English language*

This website offers a JAVA applet to calculate and display 2D L-systems online. No installation is needed. The source code for the application is available too.

http://www.javaview.de/vgp/tutor/lsystem/PaLSystem.html

*The pages of Charles Vassalo – French and English language*

This website focuses on the Markus-Lyapunov fractals with a minor component on the Mandelbrot set. In addition to the explanation of the algorithms, the website offers a gallery of wonderful designed artistic manipulation of the original fractal images. So this site is building a bridge to arts. The website is completely bilingual in English and French language.

http://perso.wanadoo.fr/charles.vassallo/index.html

*L-Systems – English language*

This is an introduction to L-Systems. The text explains L-Systems and how to use the software Fractint for creation.

http://parallel.hpc.unsw.edu.au/complex/tutorials/tutorial2.html

*Les fractales – French language*

An introduction to fractals from the type of the Mandelbrot set. There are many references to Benoit Mandelbrot. There are also many links to other sites.

http://www.csvt.qc.ca/patriotes/pei/travaux/fractales/fractales.html

*UCBLogo – English language*

The University of California in Berkley has a LOGO version ready for free download. It is available for UNIX/LINUX, Windows and Mac.

http://www.cs.berkeley.edu/~bh/logo.html

*XLOGO – English, French and Spanish language*

This is a free JAVA implementation of LOGO. Due to the JAVA implementation, this LOGO version runs on all platforms. The Koch coast line is included in the documentation as an example.

http://xlogo.free.fr/

# 2. Projects and activities

*Fractint image contest – English language*

The web page of fractal contests driven by the developers of Fractint. Unfortunately it is not sure if the contest is to be continued. The gallery of images is nevertheless impressive.

http://www.fractalartcontests.com/index.html

*Fractal Art Museum Enterprise – English language*

This web site is an entry to activities and background material around fractals. There are several contests in which pupils can participate. The background material contains for example descriptions of the formulas behind fractals and a database of ready to use formulas.

http://www.wack.ch/fame/artists/kod101.html

*Xplora expert talks*

Xplora, the European Science Education Gateway, is shceduling online discussions with experts in various fields of science. Fractals is one of the forthcoming topics. Teachers interested in a talk with an expert should go to the webpage and register for the chat.

http://www.xplora.org/chats

## 3.Teacher training opportunities

## About Xplora Thematic Dossiers

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