

The earth's radius by Eratosthenes

GeoGebra as a tool for visualisation

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Introduction

This description serves a real measurement of the earth's radius according to Eratosthenes of Cyrene, who applied this method for the first time about 206B.C. GeoGebra like GEONExT are dynamic geometry tools and I would like to show in this presentation, how such a software can be used to visualize the method and calculate the results in this activity. You need a partner school, ideally located on the same longitude. But here you can read about compensating in case you do not find a partner school on the same longitude.

The Problem

The everyday life experience seems to show that the earth is flat. It took a long time of scientific work, until humans had the insight that the earth is a bowl moving around the sun, explaining many natural phenomena in an easy way. It was at the end of the 20th century, that people could get a real photo of that bowl (Fig. 1).

We will start watching a video and then use GeoGebra to analyse how to get the results of a project run.

The Video

The video on Xplora-Knoppix demonstrates the use of the Tellurium.

From the Tellurium to GeoGebra

In the video you have seen the fading from the Tellurium to the GeoGebra construction. Now we will have a closer look at the construction in Fig. 2.



Figure 1: The earth seen from Apollo 17 (Image: Wikipedia).

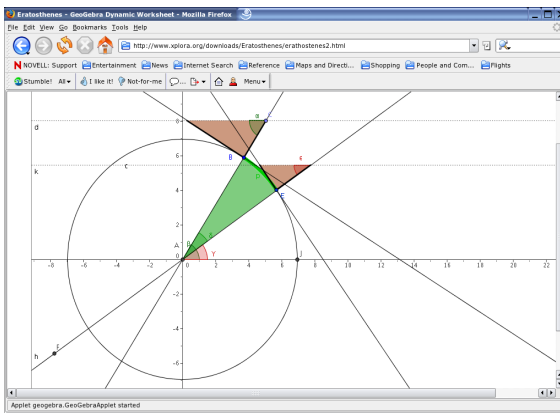


Figure 2: The GeoGebra construction

From right we see the parallel sun rays coming in. In two locations we have shadow sticks placed, which draw a shadow on the respective horizontal plane (We “see” the earth as a plane). From the length of the shadow and that of the shadow sticks we can calculate the geographical height angle.

Be l the length of the shadow, h the height of the shadow stick, then we get for the location B:

$$\alpha = \beta = \arctan\left(\frac{l_B}{h_B}\right) \quad \text{and for location E:}$$

$$\epsilon = \gamma = \arctan\left(\frac{l_E}{h_E}\right) \cdot$$

The angles β and γ are the geographical heights for both places. The difference of these angles gives the the angle $\delta = |\alpha - \gamma|$, from which we can calculate the earth's radius, if the distance p of both locations on the same meridian is known from the readings of a map:

$$r = \frac{360^\circ}{2 * \pi * \delta} * p$$

Equation 1: The
radius of earth
according to
Eratosthenes

To receive the result of equation 1, the most critical part is calculating the difference angle δ from the two geographical heights, which need the knowledge of trigonometric functions for calculation. This is no problem at higher levels, but for lower secondary classes this is a problem. In earlier versions of the Eratosthenes project, students have drawn corresponding rectangular triangles and measured the angles. This is now much easier with the help of a dynamic geometry program, but still the principle of scale true drawings is kept.

In Figure 3 we have the shadow in one location. With this application, the shadow angle of the location can be measured:

1. Draw point E, until the height h corresponds the height of the shadow stick.
2. Draw point B, until the length l corresponds the length of the shadow.
3. Read the shadow angle.

During the course of the project, the participating classes will exchange their measured geographical height angle, the height h of the shadow stick and the length l of the shadow. From these data, pupils calculate the earth's radius by using pairs of two locations.

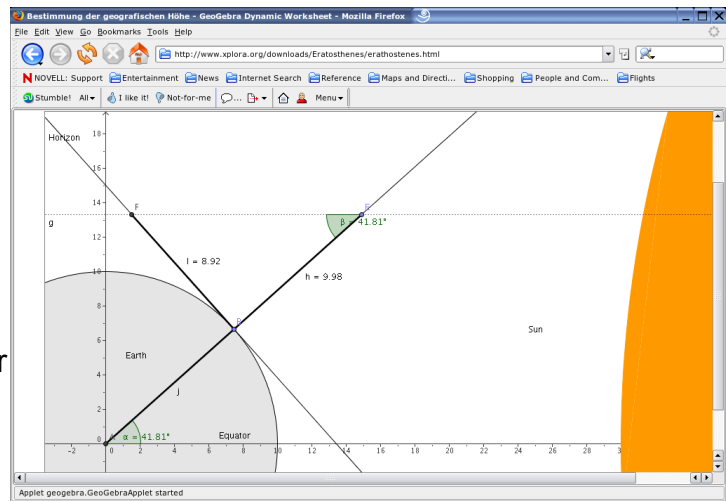


Figure 3: Construction of the geographical height

Figure 4 shows, how to use the data of two locations to get the distance p on the same meridian:

- Choose one meridian on the globe, somewhere in between your location and the location of your partner school.
- Move from your location to the selected meridian keeping the latitude.



Figure 4: Measuring the distance between two locations on the same longitude (Map from http://www.lib.utexas.edu/maps/europe/europe_ref_2005.pdf)

- Repeat the same procedure for your partner schools location.

<http://www.xplora.org>

- Look for the names of cities near the location of the two marked locations or try to read the coordinates. Be as precise as possible, as this result determines the quality of your project.
- Measure the distance on a map by measuring the distance in cm and then calculate the distance in reality (p) by using the scale factor.
- Calculate r by using equation 1 and the values of p and δ .
- Compare to the known values and calculate the relative error of the result.
- Finally prepare a description of the project and publish in the Eratosthenes community of Xplora.